Augmenting Density Matrix Renormalization Group with Clifford Circuits

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Nov 19, 2024

Refs:
1. Xiangjian Qian, Mingpu Qin, Chin. Phys. Lett. 40, 057102 (2023) (Express Letter)
2. Xiangjian Qian, Jiale Huang, Mingpu Qin, Phys. Rev. Lett. 133, 190402 (2024)
3. Xiangjian Qian, Jiale Huang, Mingpu Qin, arXiv:2407.03202 (2024)
4. Jiale Huang, Xiangjian Qian, Mingpu Qin, arXiv:2409.16895 (2024)
5. Xiangjian Qian, Jiale Huang, Mingpu Qin, arXiv:2410.15709 (2024)

The 1st workshop on many-body quantum magic, 2024

Outline

- DMRG for 2D system
- FA-MPS = MPS + Disentanglers
- CA-MPS = MPS + Clifford circuits
- Non-stabilizerness Entanglement Entropy
- Conclusion and perspectives

Success of DMRG in 1D

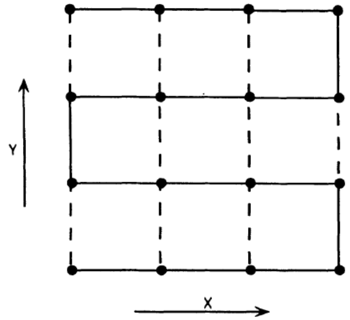
TABLE I. Ground-state energies per site of infinite $S=\frac{1}{2}$ and $S=1$ antiferromagnetic Heisenberg chains. The exact Bethe-ansatz result for the energy of the $S=\frac{1}{2}$ chain is $-\ln 2+\frac{1}{4}=-0.443\,147\ldots$, and m is the number of states kept in block A (counting a triplet as three states, etc.). Results labeled ∞ are obtained from a linear extrapolation to $P_m \to 1$. Monte Carlo results are taken from Refs. [7] and [5].

m	$S=\frac{1}{2}$ $E_0-E_0^{\text{exact}}$	$S=\frac{1}{2}$ $1-P_m$	$S=1$ $-E_0$	$S=1$ $1-P_m$
16	5.8×10^{-5}	8.0×10^{-6}	1.401 089	4.8×10^{-5}
24	1.7×10^{-5}	1.9×10^{-6}	1.401 380	1.6×10^{-5}
36	7.8×10^{-6}	9.0×10^{-7}	1.401 437	6.6×10^{-6}
44	3.2×10^{-6}	3.6×10^{-7}	1.401 476	1.1×10^{-6}
∞	1.9×10^{-7}		1.401 484(2)	
MC	$\sigma=5\times10^{-4}$		1.401 5(5)	

Steven R. White, Phys. Rev. Lett. 69, 2863 (1992)

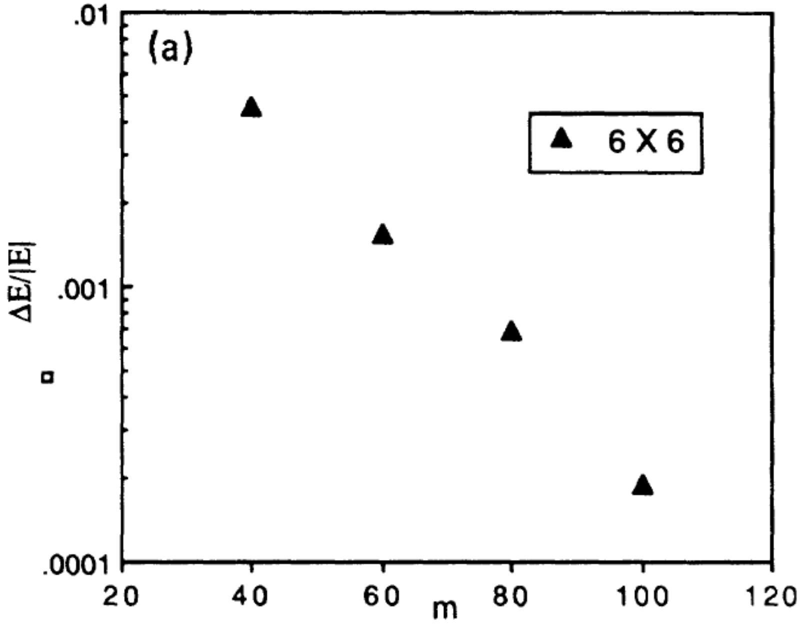
Failure of DMRG in 2D

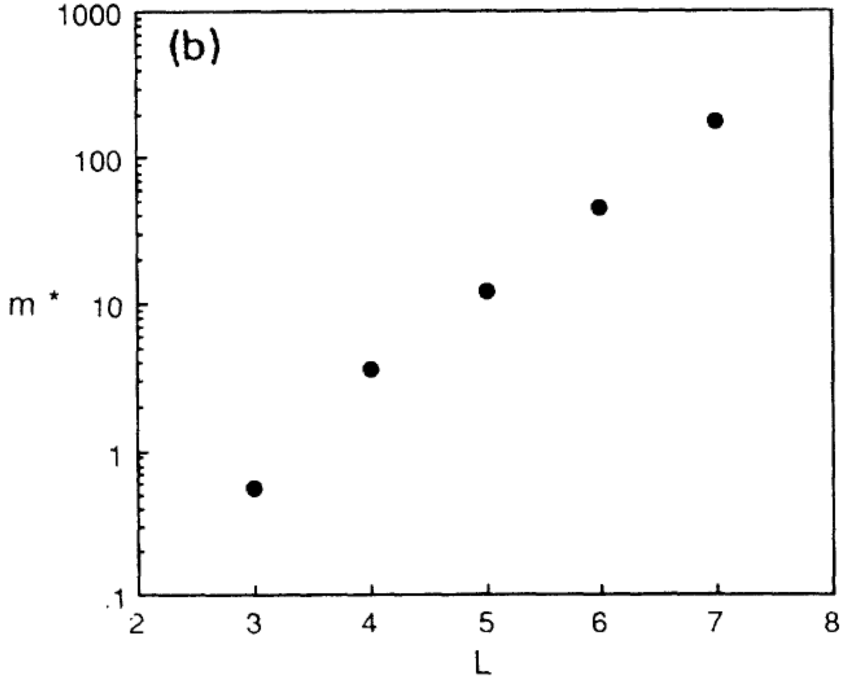
2D free spinless fermion



Map to 1D

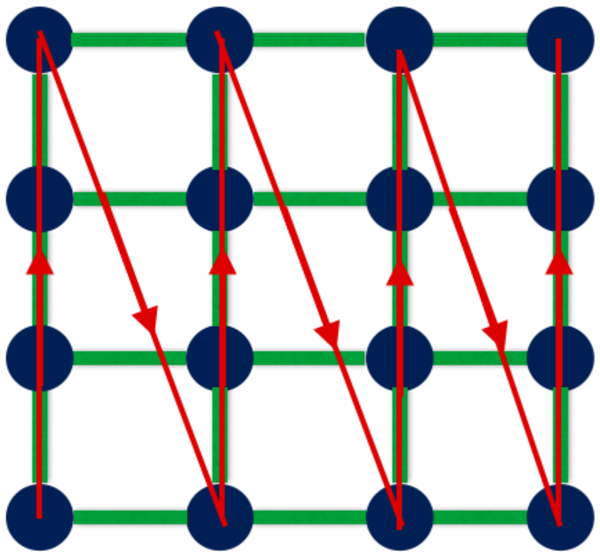
$$\Delta E = Ce^{-m/m^*}, \quad m^* = \alpha^L/A$$

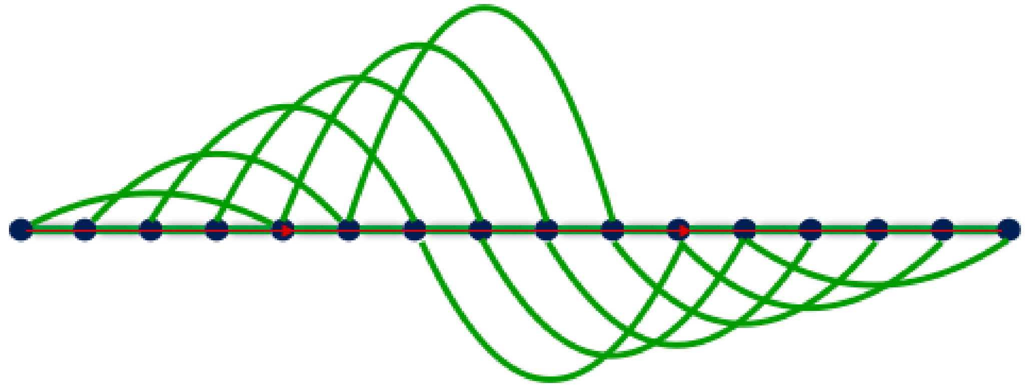


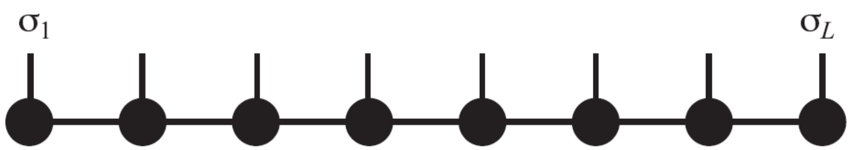


Shoudan Liang and Hanbin Pang, Phys. Rev. B 49, 9214 (1994)

Failure of DMRG in 2D







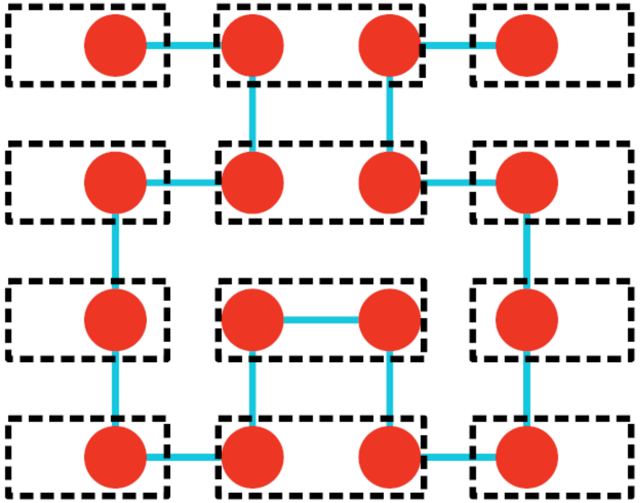
Matrix Product States (MPS)

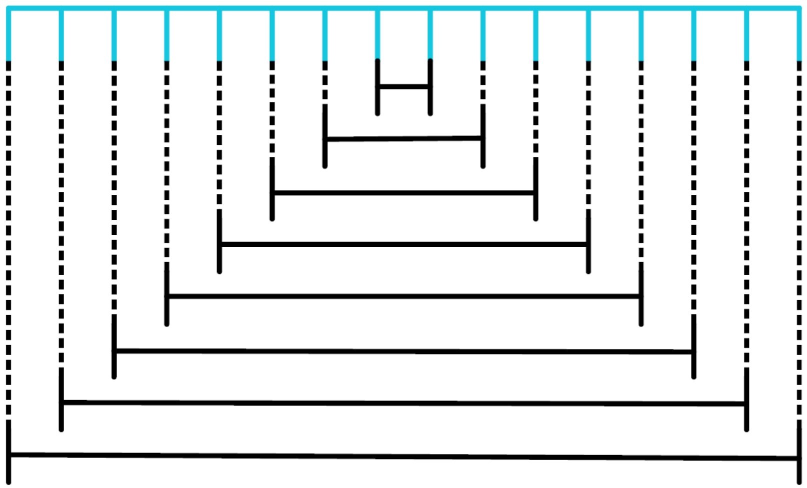
Area law for 2D ground state: **EE~L**

DMRG/MPS can support: **EE < log(D)**

Can we increase entanglement in DMRG while keeping the low cost?

Fully-augmented MPS (FA-MPS)





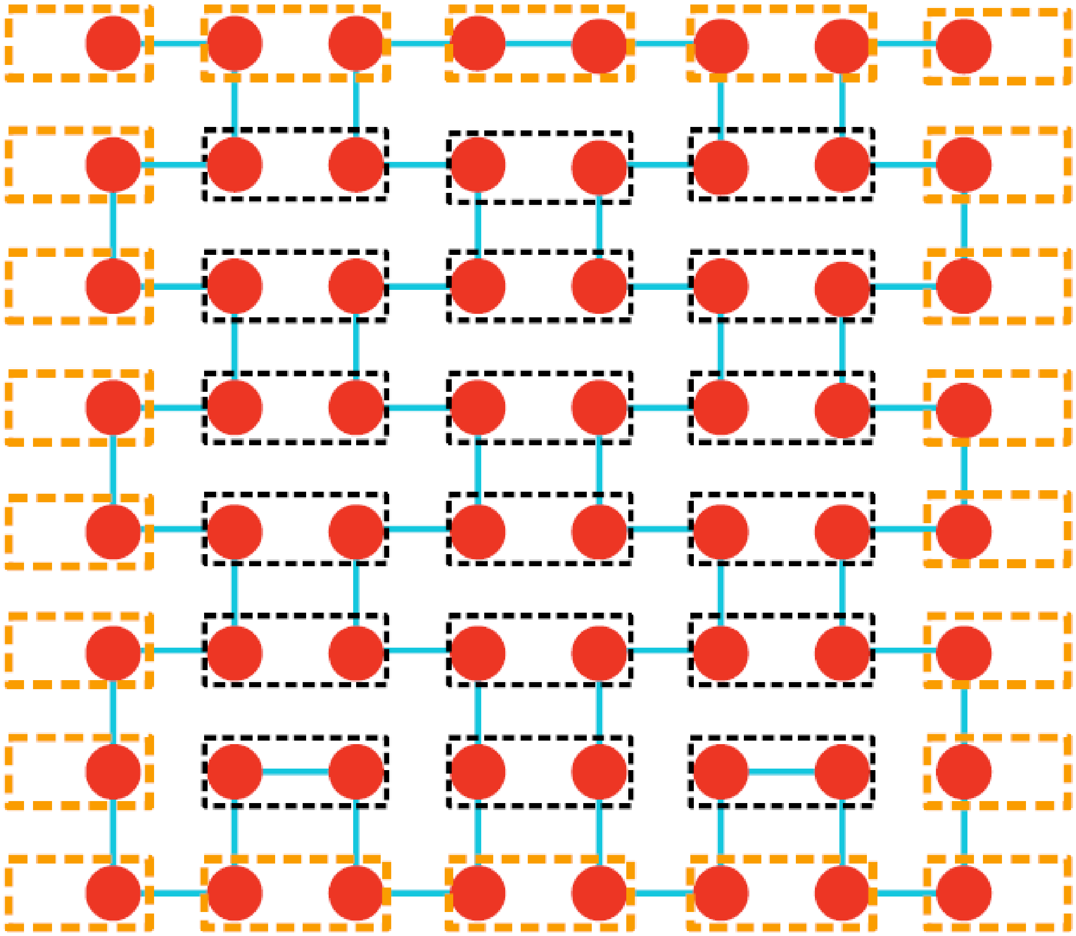
Principles:

- ☐ No overlap: easy to be optimized
- ☐ More disentanglers at bottleneck

☐ Cost: O(d^4) * DMRG

Xiangjian Qian, Mingpu Qin, Chin. Phys. Lett. 40, 057102 (2023) (Express Letter)

FA-MPS can support area-law-like entanglement

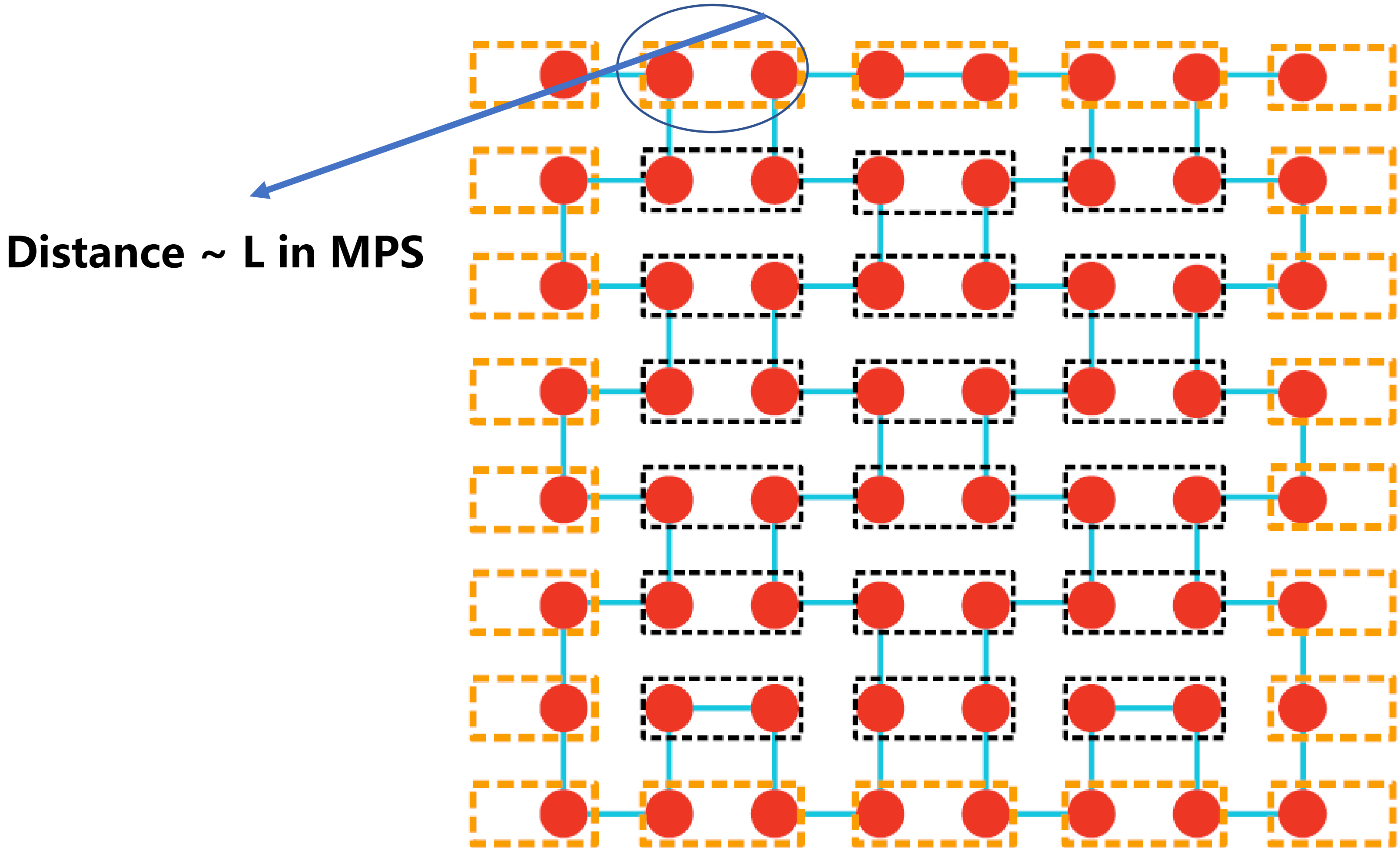


EE FA-MPS can support:

$$L \ln(d^2)$$

Xiangjian Qian, Mingpu Qin, Chin. Phys. Lett. 40, 057102 (2023) (Express Letter)

FA-MPS can support area-law-like entanglement



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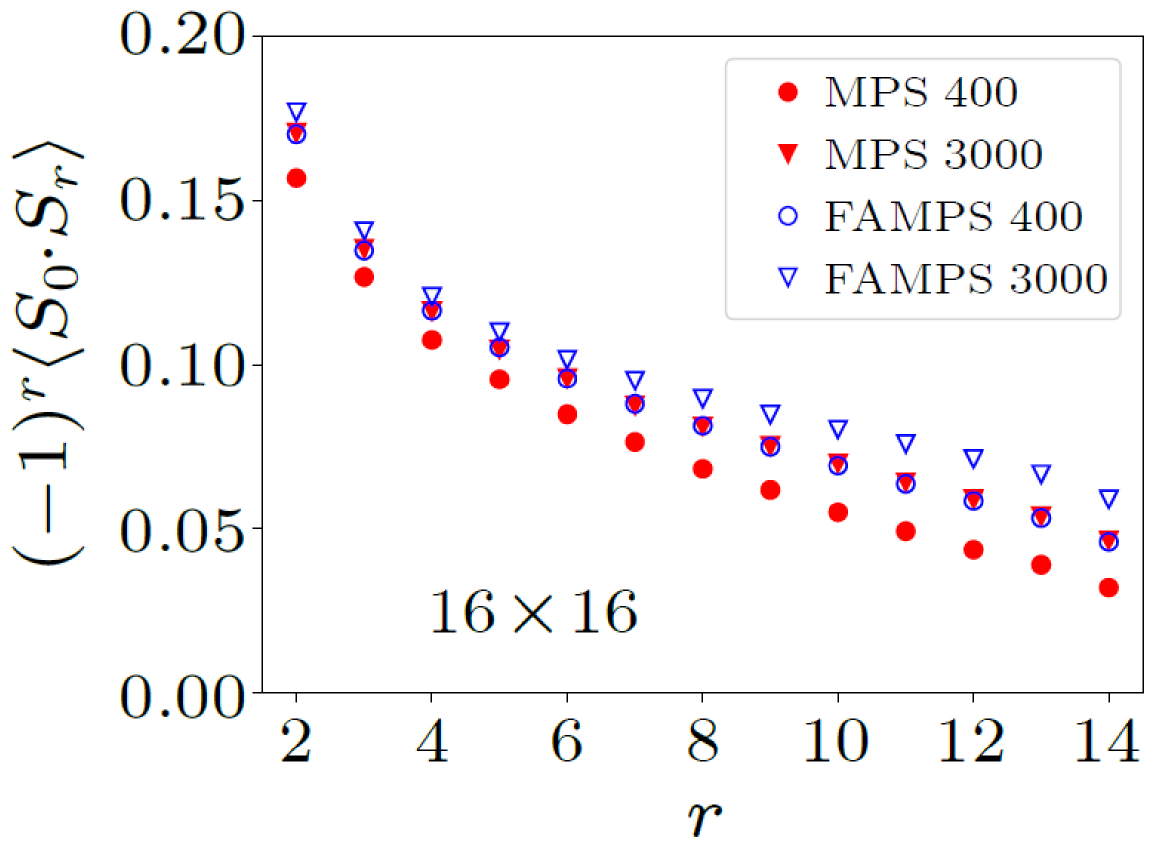
$$L \ln(d^2)$$

Xiangjian Qian, Mingpu Qin, Chin. Phys. Lett. 40, 057102 (2023) (Express Letter)

Benchmark results



2D Heisenberg model: OBC

FA-MPS **D** ~ MPS **10D**

Xiangjian Qian, Mingpu Qin, Chin. Phys. Lett. 40, 057102 (2023) (Express Letter)

Clifford Circuits can support large entanglement

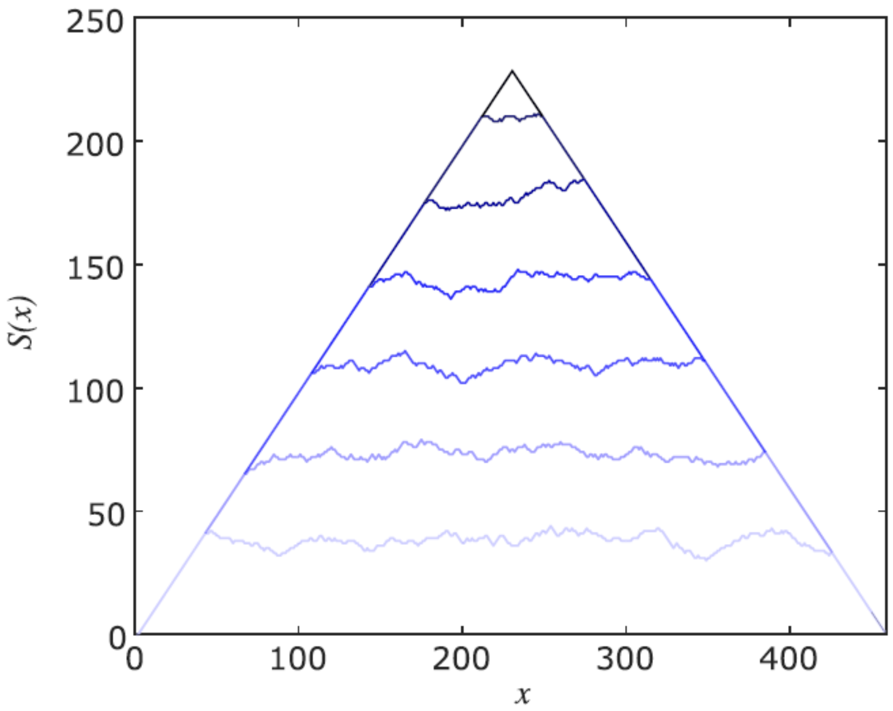


FIG. 15. The von Neumann entropy $S(x, t)$ for a system of length $L = 459$, as a function of x, for several successive times ($t = 340, 690, 1024, 1365, 1707, 2048$, and 4096), in the Clifford evolution. This shows that the state evolves from a product state to a near-maximally-entangled one. Prior to saturation the entanglement displays KPZ-like stochastic growth. $S(x, t)$ is in units of log 2.

Adam Nahum, Jonathan Ruhman, Sagar Vijay, and Jeongwan Haah, Phys. Rev. X 7, 031016 (2017)

How can we take advantage of Clifford Circuits?

Matrix Product States/Tensor Network States

Stabilizer States

Limited entanglement

DMRG “solves” one dimensional quantum systems

Large entanglement

Clifford circuits: ***non****universal* for quantum computing

R. V. Mishmash, T. P. Gujarati, M. Motta, H. Zhai, G. K.-L. Chan, and A. Mezzacapo, Journal of Chemical Theory and Computation 19, 3194 (2023),
Sergi Masot-Llima, Artur Garcia-Saez, arXiv:2403.08724 (2024)
G. Lami, T. Haug, and J. D. Nardis, arXiv:2404.18751 (2024)

Clifford gates preserve the Pauli string

Property of Clifford gates: preserve the Pauli string structure

$$HXH^\dagger = Z \qquad SXS^\dagger = Y$$

$$HZH^\dagger = X \qquad SZS^\dagger = Z$$

CNOT combinations

P	CNOT P CNOT†
$X \otimes I$	$X \otimes X$
$I \otimes X$	$I \otimes X$
$Z \otimes I$	$Z \otimes I$
$I \otimes Z$	$Z \otimes Z$

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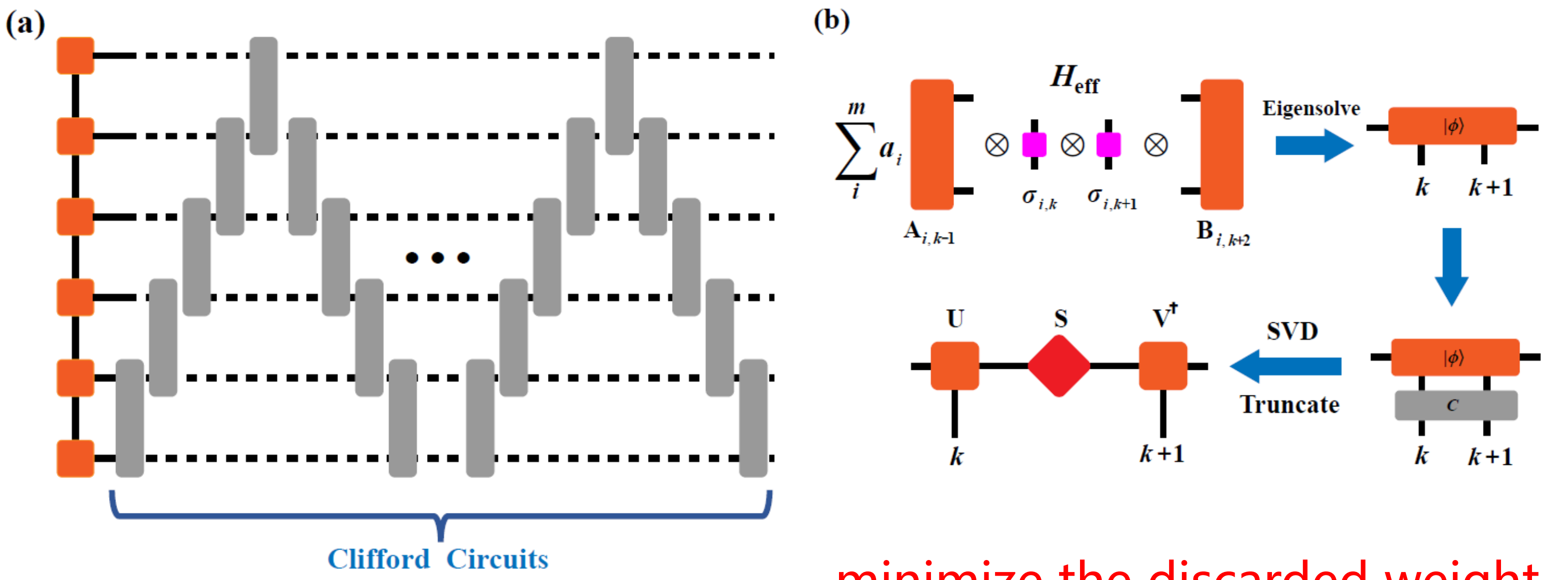
$I \otimes Z$	$Z \otimes Z$

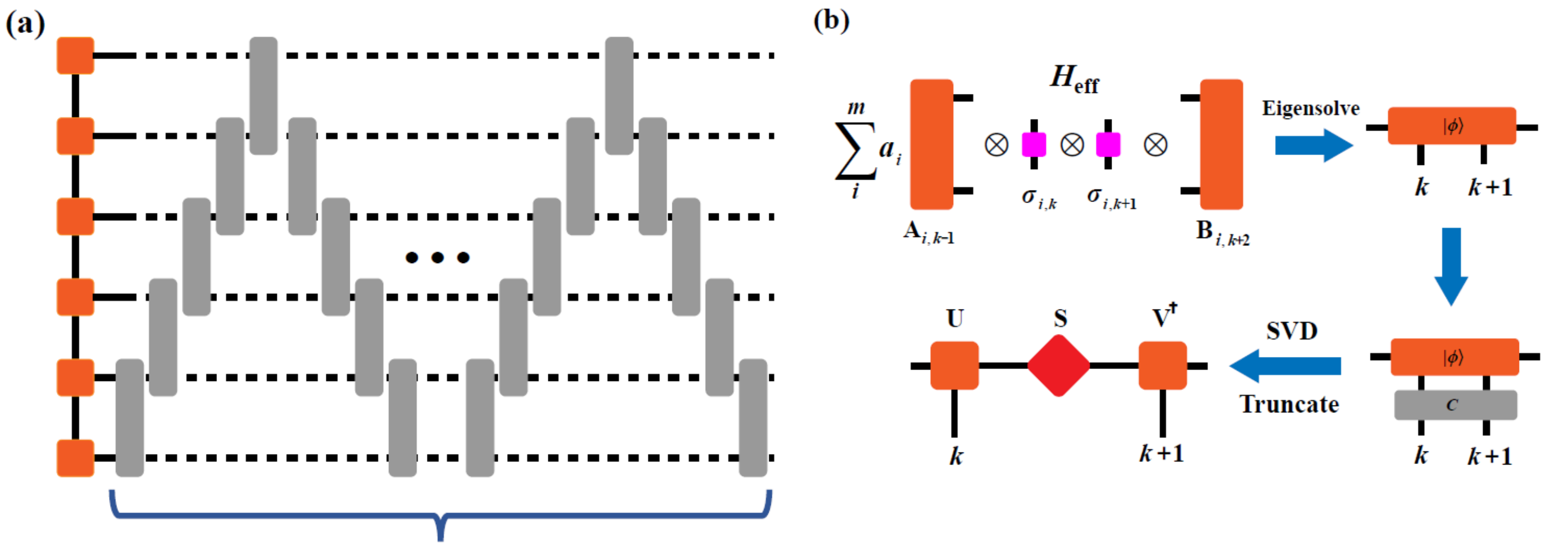
But $$TXT^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\frac{\pi}{4}} \end{bmatrix} = \begin{bmatrix} 0 & e^{-i\frac{\pi}{4}} \\ e^{i\frac{\pi}{4}} & 0 \end{bmatrix} \notin \mathbf{P}_1$$

Clifford Circuits Augmented MPS



minimize the discarded weight

Clifford Circuits Augmented MPS



minimize the discarded weight

$$H' = CHC^\dagger = \sum_i^m a_i C P_i C^\dagger$$

$$= \sum_i^m a_i \sigma_1 \otimes \cdots \sigma_{k-1} C \sigma_{i,k} \otimes \sigma_{i,k+1} C^\dagger \sigma_{i,k+2} \otimes \cdots \sigma_N$$

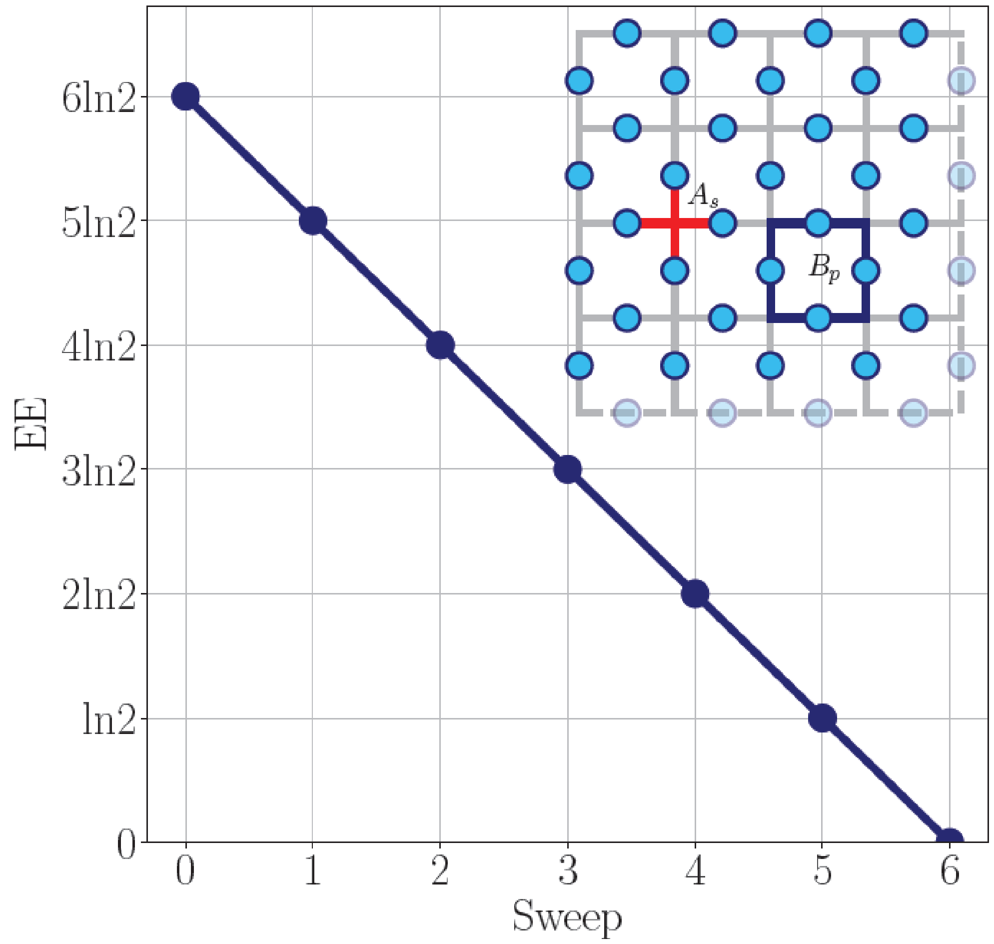
$$= \sum_i^m a_i' \sigma_1 \otimes \cdots \sigma_{k-1} \sigma'_{i,k} \otimes \sigma'_{i,k+1} \sigma_{i,k+2} \otimes \cdots \sigma_N$$

$$H_{\text{eff}}|\phi\rangle = E_g|\phi\rangle \longrightarrow C H_{\text{eff}} C^\dagger C|\phi\rangle = E_g C|\phi\rangle$$

Under Clifford Circuits, # of interaction terms remains unchanged

Xiangjian Qian, Jiale Huang, Mingpu Qin, Phys. Rev. Lett. 133, 190402 (2024)

Results for CA-MPS



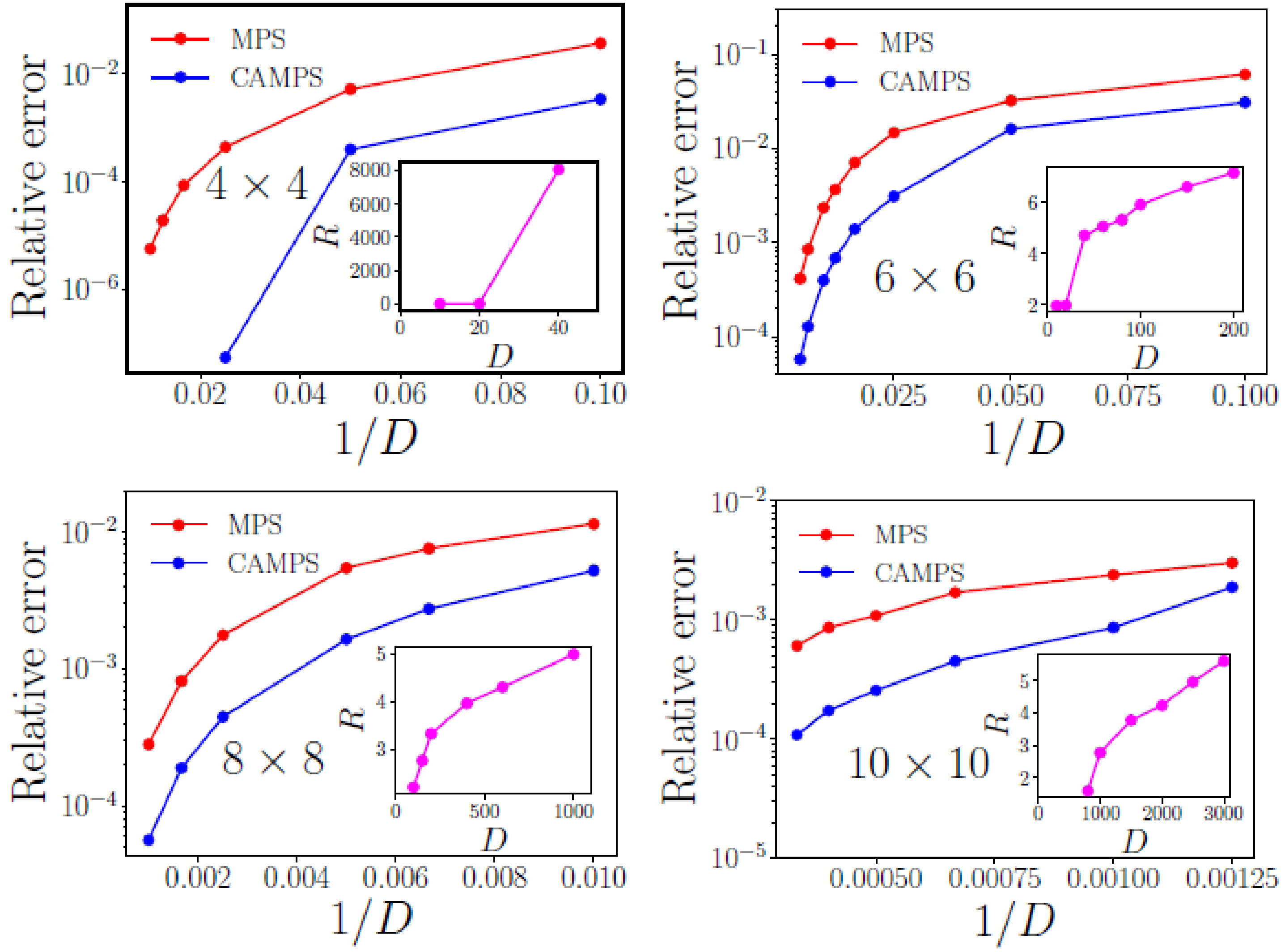
$$A_s = \prod_{i\in s} \sigma_i^x \qquad B_p = \prod_{i\in p} \sigma_i^z$$

$$H_{\text{Toric}} = -\sum_s A_s - \sum_p B_p$$

- EE = (2L − 2)ln2 for toric code model
- Every step decreases ln2
- EE = 0 after convergece

Xiangjian Qian, Jiale Huang, Mingpu Qin, arXiv:2409.16895 (2024)

Results for CA-MPS



Heisenberg OBC

Xiangjian Qian, Jiale Huang, Mingpu Qin, Phys. Rev. Lett. 133, 190402 (2024)

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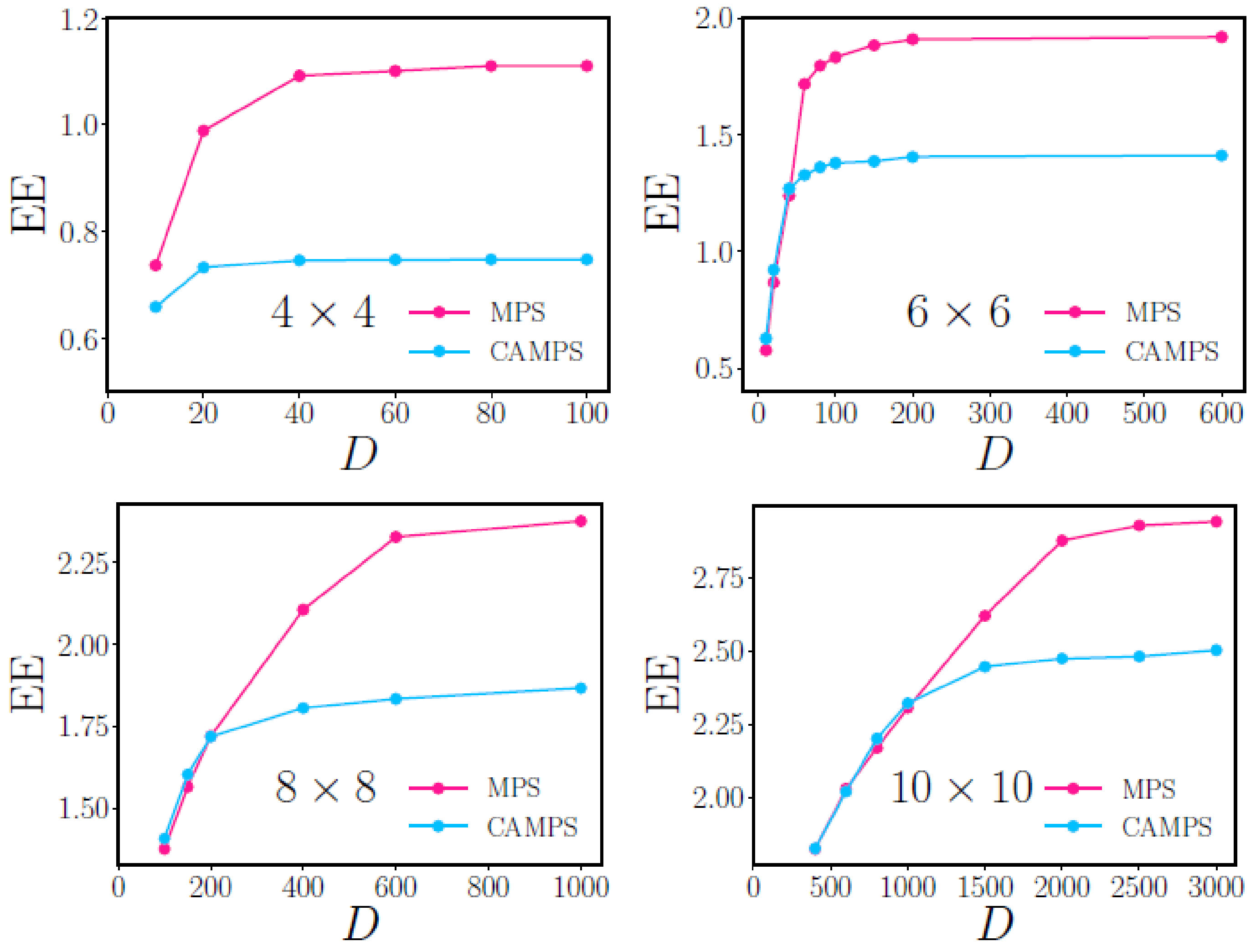

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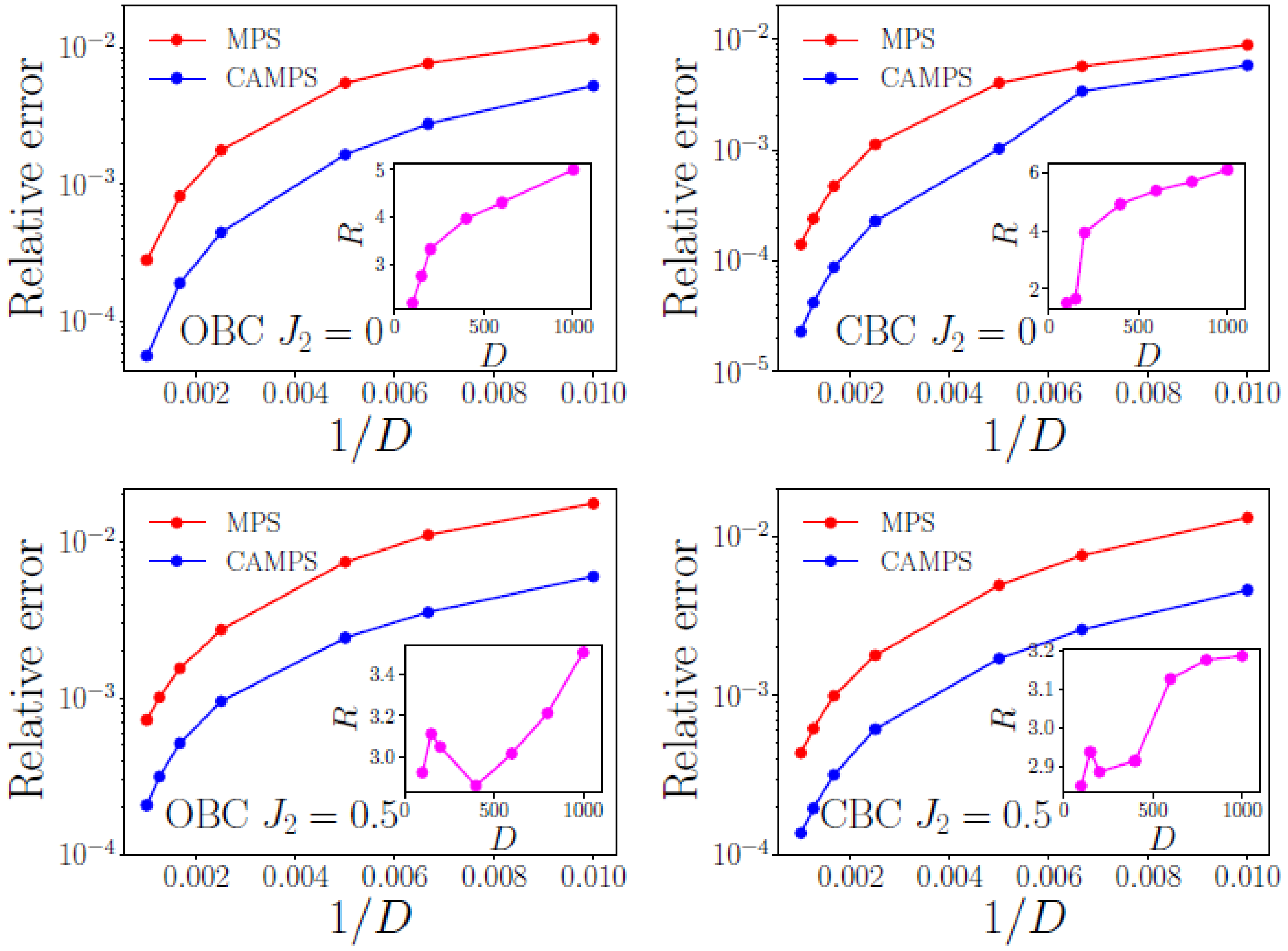
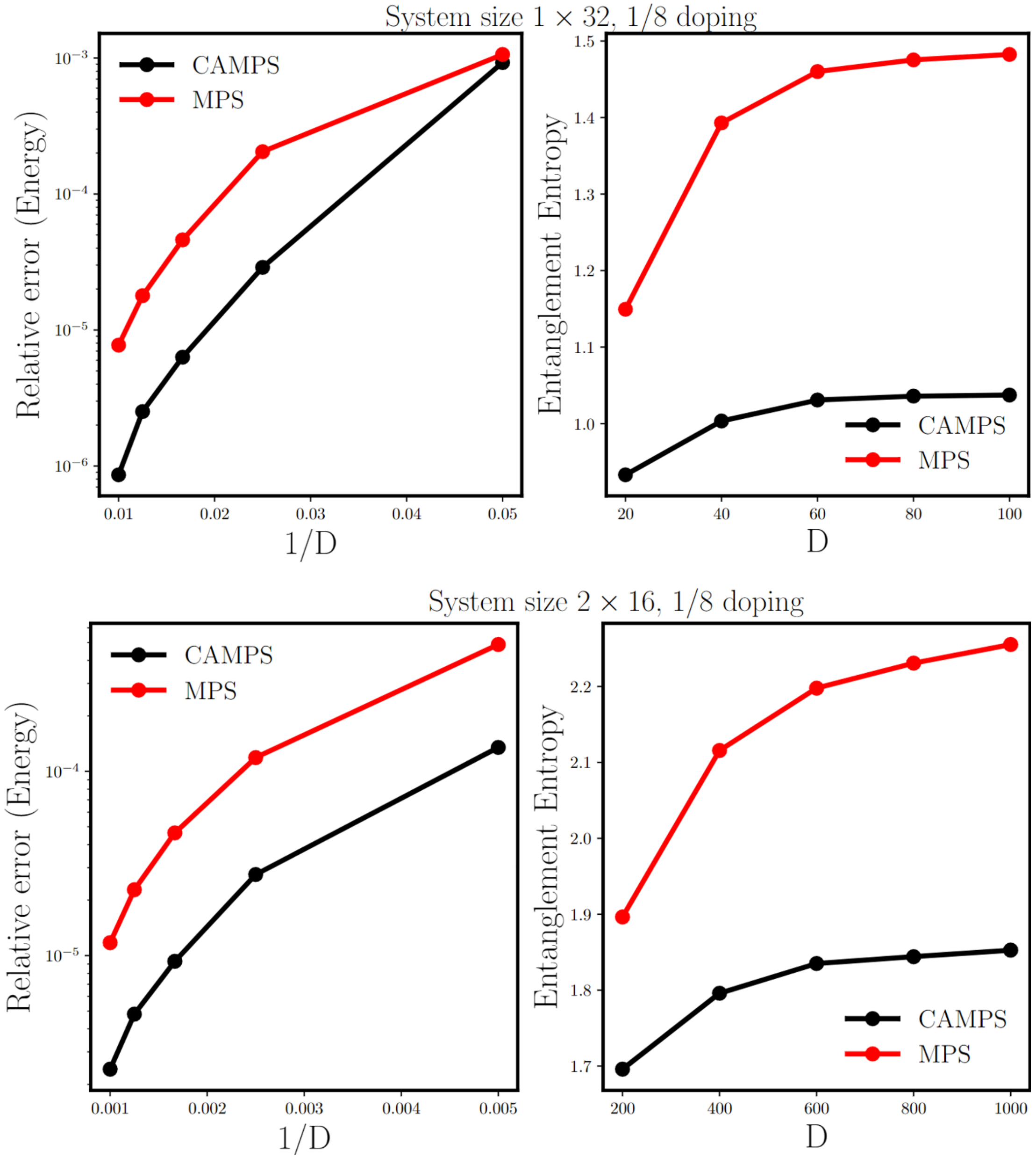


J1-J2 Heisenberg

8 * 8

Xiangjian Qian, Jiale Huang, Mingpu Qin, Phys. Rev. Lett. 133, 190402 (2024)

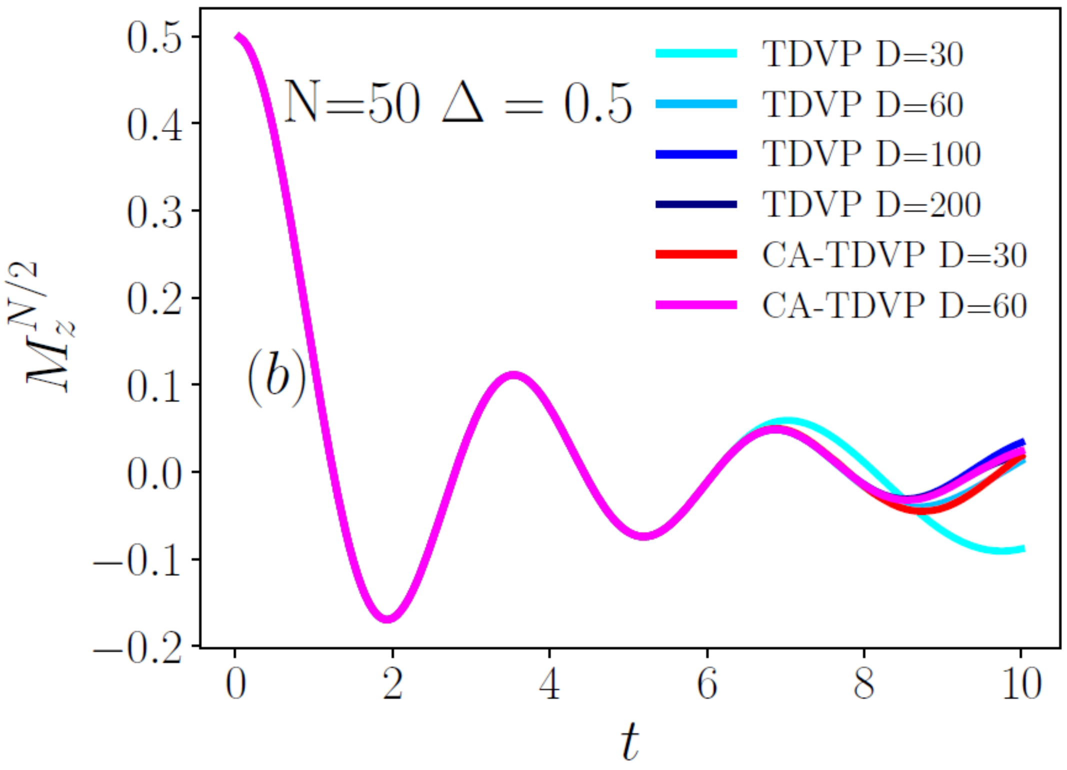
Results for CA-MPS



Hubbard model, U=8, unpublished preliminary results

TDVP+Clifford circuits: dynamics

XXZ chain



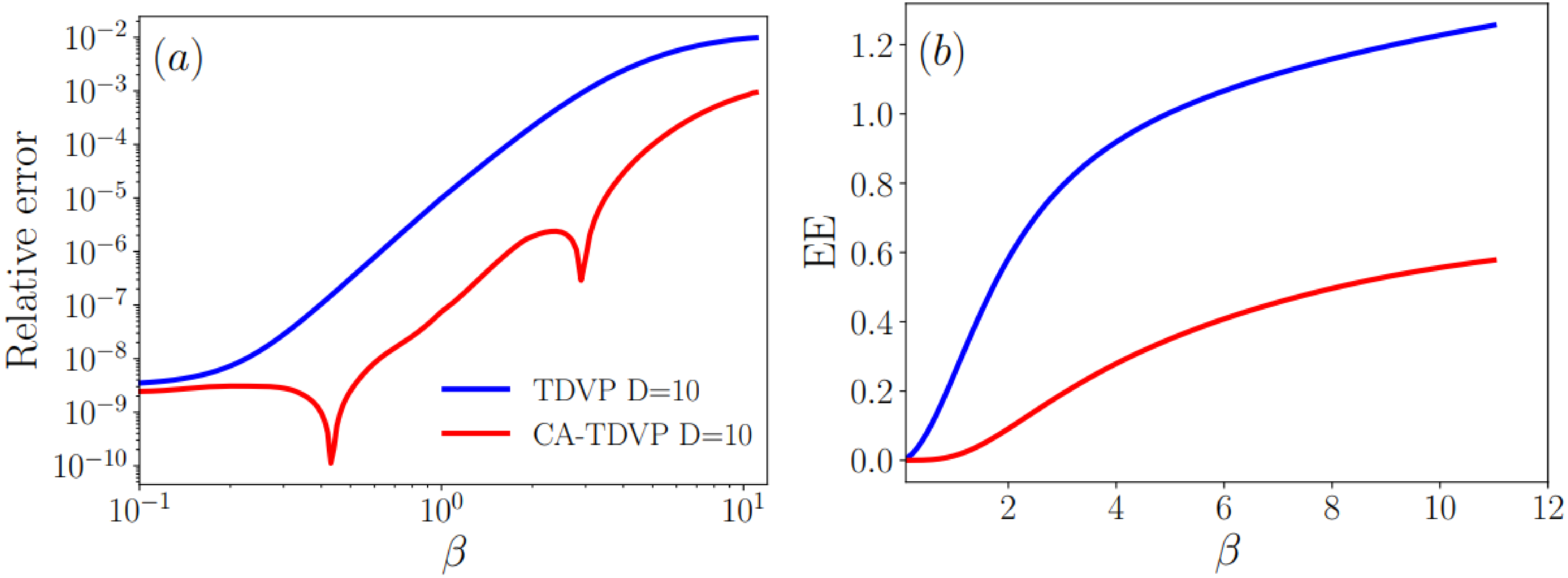
Initial state:

$|\uparrow\downarrow\uparrow\downarrow\cdots\uparrow\downarrow\uparrow\downarrow\rangle$

Xiangjian Qian, Jiale Huang, Mingpu Qin, arXiv:2407.03202 (2024)
See also: A. F. Mello, A. Santini, G. Lami, J. D. Nardis, and M. Collura, arXiv:2407.01692

TDVP+Clifford circuits: finite temperature

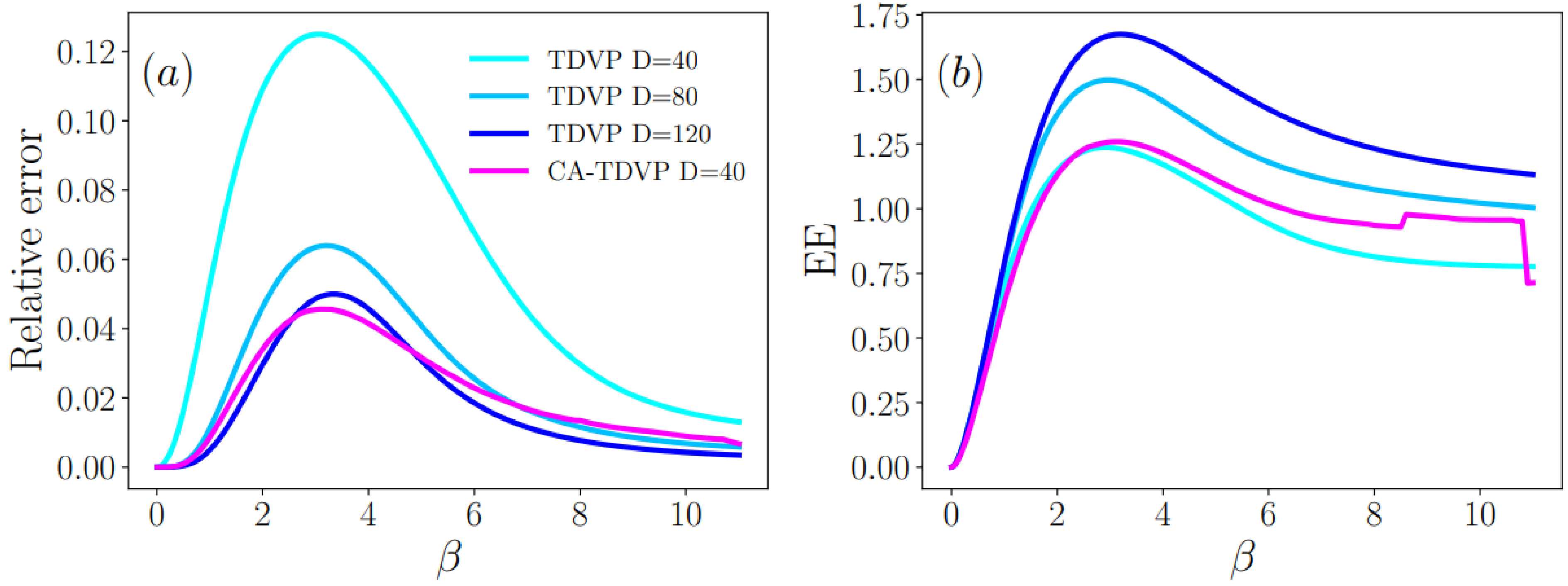
Heisenberg chain: energy



Xiangjian Qian, Jiale Huang, Mingpu Qin, arXiv:2410.15709 (2024)

TDVP+Clifford circuits: finite temperature

2D J1-J2, J2/J1 = 0.5, 4 * 4, energy

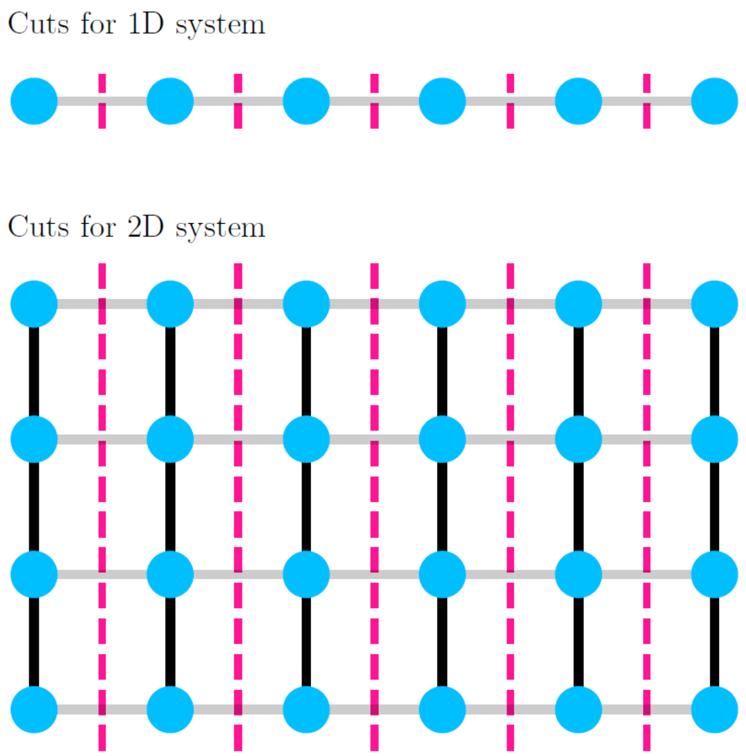


Xiangjian Qian, Jiale Huang, Mingpu Qin, arXiv:2410.15709 (2024)

Non-stabilizerness Entanglement Entropy

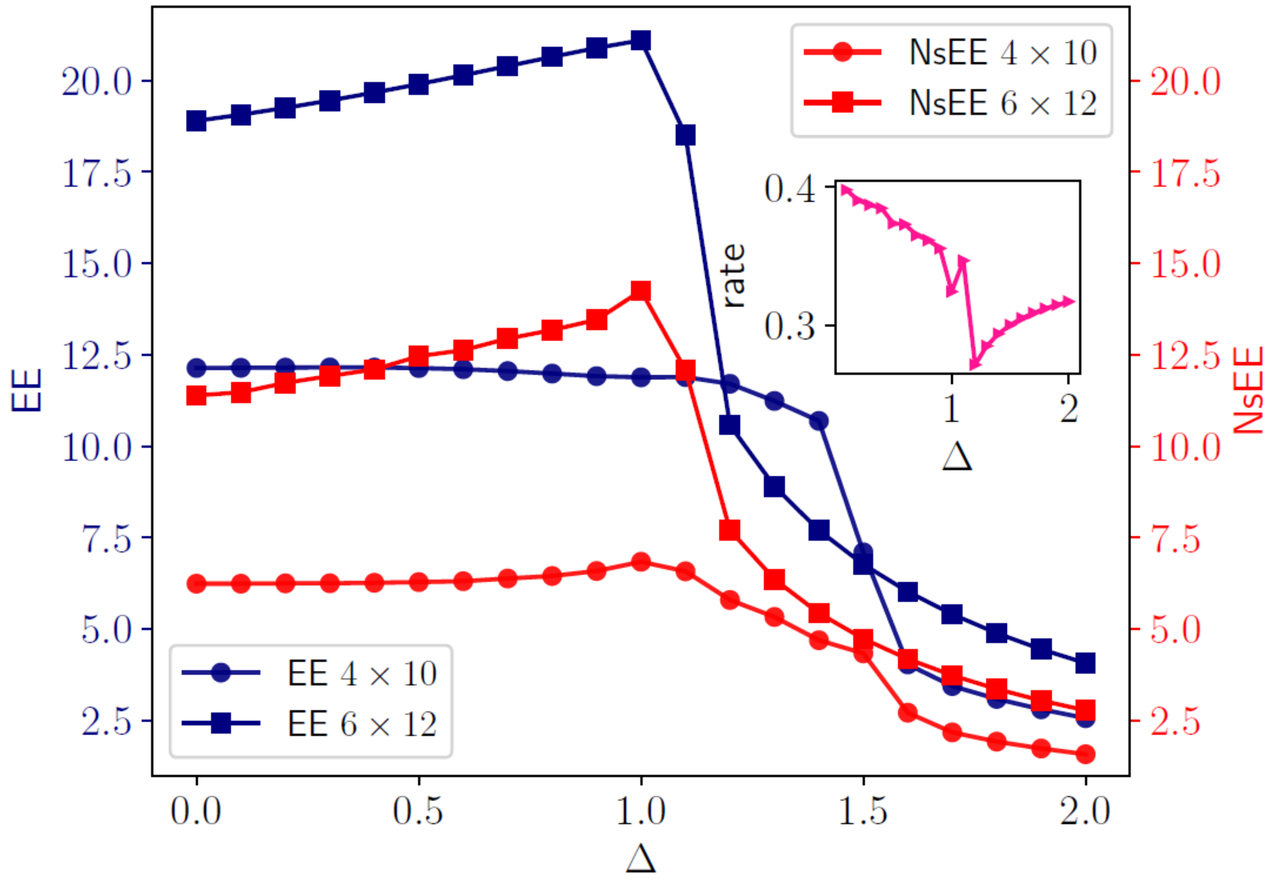
$$\mathrm{NsEE}(|\psi\rangle) = \min_{\{\mathcal{C}\}} \sum_{\text{cuts}} \mathrm{EE}(\mathcal{C}|\psi\rangle)$$

1. NsEE ≥ 0 for any pure state.
2. NsEE is zero for stabilizer states, $\mathrm{NsEE}(|Stab.\rangle) = 0$.
3. Stability under Clifford operations, i.e., $\mathrm{NsEE}(\mathcal{C}|\psi\rangle) = \mathrm{NsEE}(|\psi\rangle)$.
4. NsEE is also additive, $\mathrm{NsEE}(|\psi\rangle_A \otimes |\psi\rangle_B) = \mathrm{NsEE}(|\psi\rangle_A) + \mathrm{NsEE}(|\psi\rangle_B)$.



Jiale Huang, Xiangjian Qian, Mingpu Qin, arXiv:2409.16895 (2024)

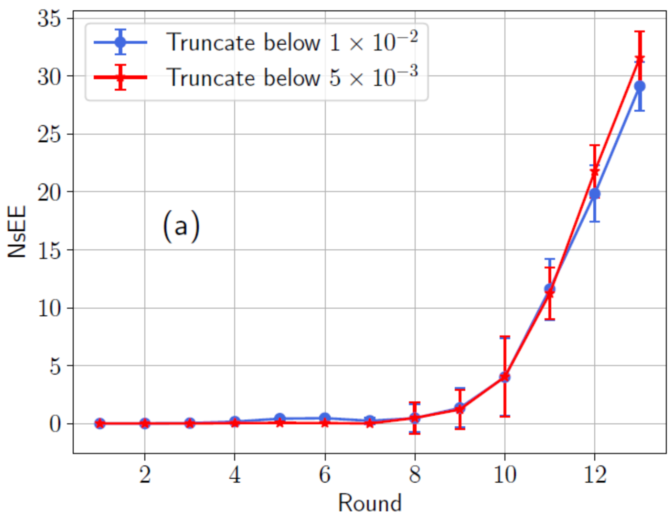
Examples: 2D XXZ

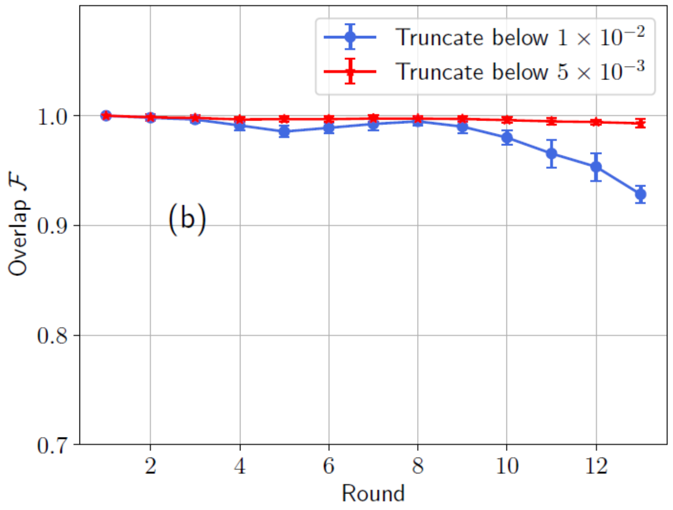


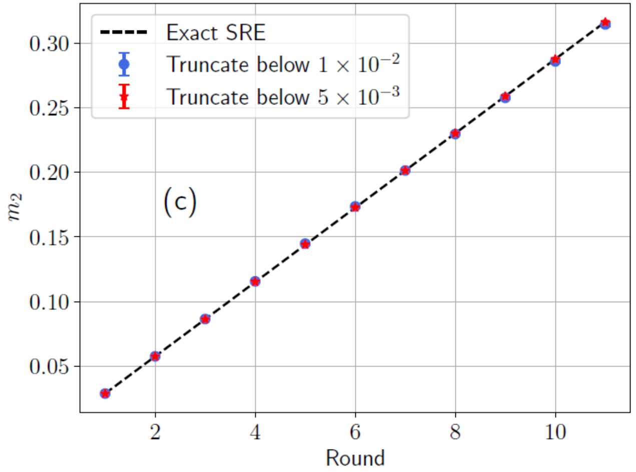
$$H_{\rm XXZ} = J \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)$$

Jiale Huang, Xiangjian Qian, Mingpu Qin, arXiv:2409.16895 (2024)

Examples: random Clifford+T circuits





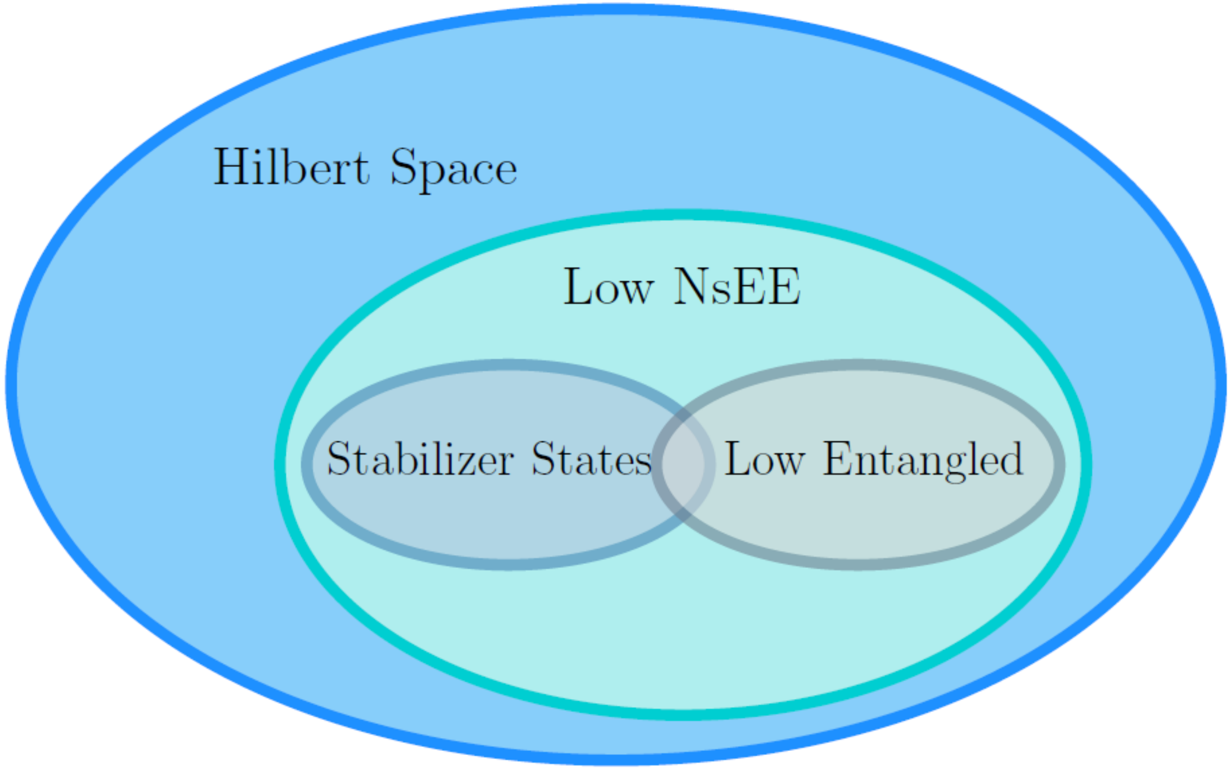


For small round (<10) of T (for a L = 20 system)

- NSEE: zero
- SRE: non-zero

Jiale Huang, Xiangjian Qian, Mingpu Qin, arXiv:2409.16895 (2024)

Hardness in the classical simulation of quantum many-body systems



Jiale Huang, Xiangjian Qian, Mingpu Qin, arXiv:2409.16895 (2024)

Discussion and perspectives

- The essence of CA-MPS is to transfer "stabilizer entropy" to Clifford circuits, reducing the burden of MPS.
- CA-MPS can be used to effectively calculate magic.
- The idea is quite versatile, could be applied to almost any MPS/PEPS related algorithm.
- NsEE can be used as a measure of hardness for classical simulation.

Discussion and perspectives

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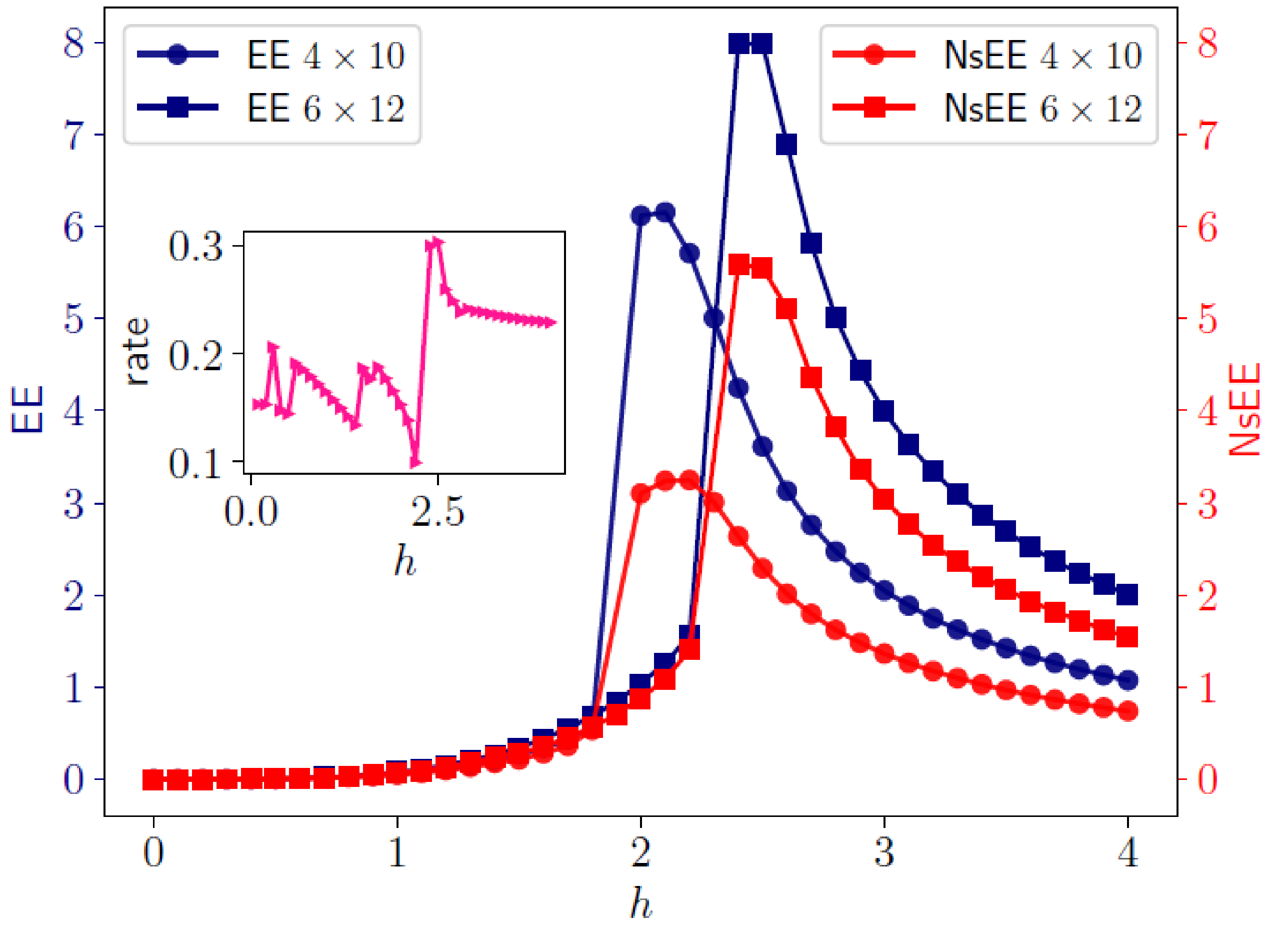
- NsEE can be used as a measure of hardness for classical simulation.

Thanks

Examples: 2D Transverse Ising



$$H_{\mathrm{Ising}} = -J\sum_{\langle i,j\rangle}\sigma_i^z\sigma_j^z - h\sum_i \sigma_i^x$$

Xiangjian Qian, Jiale Huang, Mingpu Qin, arXiv:2409.16895 (2024)

Stabilizer Renyi Entropy

$$M_n(\psi\rangle) = \frac{1}{1-n} \log \sum_{P \in P_N} \frac{\langle\psi|P|\psi\rangle^{2n}}{2^N}$$

1. $M_n \geq 0$ for any pure state and $M_n = 0$ for stabilizer state.
2. M_n is invariant under Clifford operations.
3. M_n is additive, which means it grows extensively with the system size.

$|T\rangle^{\otimes N}$ with $|T\rangle = \frac{|0\rangle + e^{i\frac{\pi}{4}}|1\rangle}{\sqrt{2}}$

$M_2 = -N \log[(1+\cos^4 \frac{\pi}{4} + \sin^4 \frac{\pi}{4})/2] > 0.$

Jiale Huang, Xiangjian Qian, Mingpu Qin, arXiv:2409.16895 (2024)